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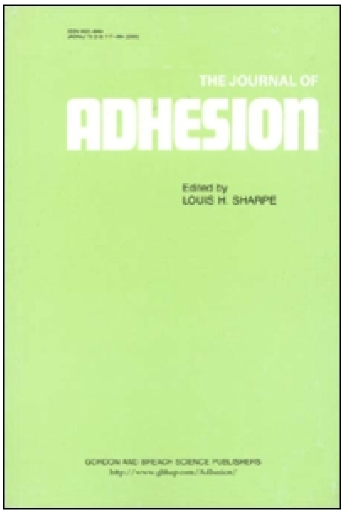

The Journal of Adhesion

Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713453635

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To cite this Article Alwar, R. S. and Nagaraja, Y. R.(1976) 'Elastic Analysis of Adhesive Butt Joints', The Journal of Adhesion, 7: 4, 279 — 287

To link to this Article: DOI: 10.1080/00218467608075059

URL: http://dx.doi.org/10.1080/00218467608075059

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J. Adhesion, 1976, Vol. 7, pp. 279–287

Printed in Scotland

Elastic Analysis of Adhesive Butt Joints

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(*Received November 15, 1974*)

The investigation of adhesive butt joints using the finite element method of solution indicates a variation of stresses through the thickness of the adhesive in contrast to the existing analyses which assume zero stress variation. Both plane stress and axisymmetric cases have been investigated and the effect of adhesive modulus and thickness on the stress distribution have been considered. It has been shown that there is significant variation of normal and shear stresses across the adhesive thickness especially at the adhesive–air interface.

INTRODUCTION

During the past several years adhesive joints have assumed increasing importance in a wide variety of engineering applications. The important reasons for this wide usage are that the adhesive bonding leads to joints with reduced stress concentrations and to light weight construction.

The theoretical analysis of lap joints has been attempted by many investigators,[1-6], whereas very few have attempted analyses for butt joints,[7,8] and only under restricted assumptions, like infinitely rigid adherends. Most of the analyses connected with the adhesive joint, however are based on the assumption that the stresses do not vary through the thickness of the adhesive, thus reducing the role of the adhesive to that of a transfer medium between two adherends. This assumption simplifies the analysis to a great extent and has resulted in obtaining simplified expressions for the strength of the joints.

The object of this investigation is to show that the stresses vary through the

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thickness of the adhesive. In contrast to the usual assumption of zero stress variation, it will be shown that both the normal and shear stresses vary significantly across the thickness of the adhesive especially at the edge of the joint.

In an interesting paper by Pirvics,[12]† a finite difference technique has been used to analyse adhesive joints. To show the versatility of the technique Pirvics has solved an adhesive butt joint with a void in the adhesive. The very existence of a void introduces changes in the stress pattern which may include the variation of stresses across the adhesive thickness. In the present investigation the finite element method has been used to solve the problem of a butt joint without any void. It is shown that even in this case of a complete butt joint there exists a variation of stresses across the thickness of the adhesive. The existence of stress singularity at the interface corner, and the variation of the stresses through the thickness of the adhesive, make the validity of the conventional approach to strength evaluation of adhesive joints questionable. Wooley and Carver[9] attempted to solve the problem of a single lap joint using the finite element technique, taking only two constant stress elements across the thickness of the adhesive. This may not be sufficient to take into account the stress variation across the adhesive thickness. This is especially so for sufficiently thick adhesive layers. To the authors' knowledge, the only rigorous analysis concerned with the elastic analysis of scarf joints of which butt joints form a special case is by Pethinaidu,[10] where an energy method is used to obtain the stresses in the joint with the usual assumption of absence of stress variation across the thickness of the adhesive.

FINITE ELEMENT ANALYSIS

Both plane stress and axisymmetric butt joints are analysed by the finite element technique. The physical problem is shown in Figure 1. Since the joint is symmetrical only one quarter of the joint is used in the analysis. The element configuration is shown in Figure 2. The X and Y coordinates for plane stress case are indicated in Figure 2. For the axisymmetric case the same axes would represent the R and Z axes respectively.

The assembly consists of 305 nodal points with 553 constant stress triangular elements. The density of the elements varies as indicated in Figure 2, the pattern being adopted to account for the stress gradient at adhesive–air and adhesive–adherend interfaces. The width of the adhesive is divided into 40 parts and the adhesive has been represented across its thickness by 8 layers of elements in order to study the stress variation through the thickness of the

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† The authors are grateful to Dr. L. H. Sharpe, Bell Telephone Laboratories, for bringing this paper to their attention.

adhesive. The butt joint is subjected to uniaxial tension (Figure 1). The total load on the edge has been suitably lumped at the nodes for the two cases of plane stress and axisymmetry. The geometrical boundary conditions imposed are shown in Figure 2.

The numerical results cover two values of the ratio b/η and two values of ratio, E_A/E_C. The adherend corresponds to steel with a Young's modulus of

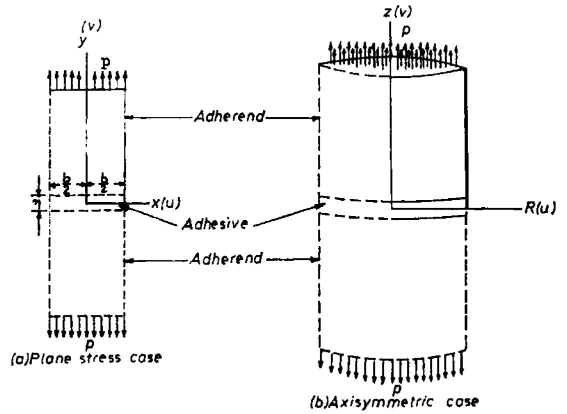

FIGURE 1 Schematic representation of joints solved.

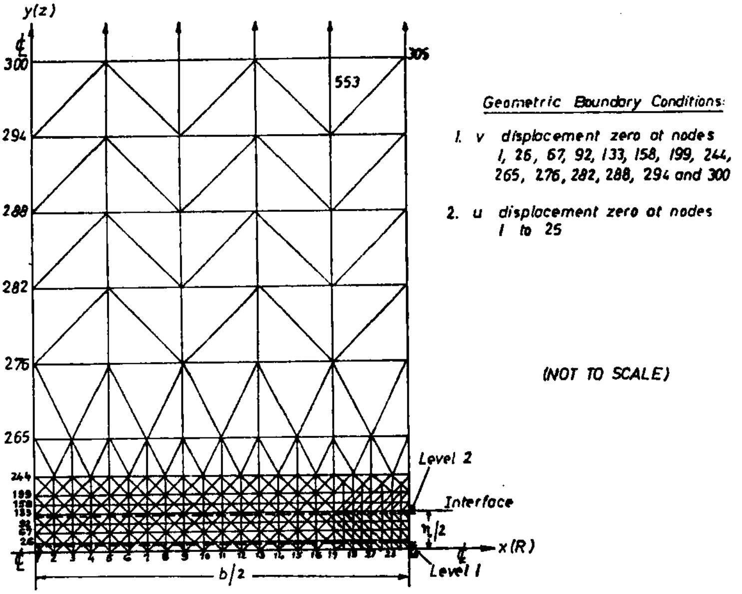

FIGURE 2 Finite element configuration.

2.0 × 10^6 kg/cm^2, and two values of Young's modulus are used for the adhesive: the value of 2.198 × 10^4 kg/cm^2 ($E_A/E_C = 91$), corresponds to a rigid epoxy and 4.395 × 10^3 kg/cm^2 ($E_A/E_C = 455$) corresponding to a high density polyethylene. In each case stress distribution along two levels within the thickness of the adhesive is evaluated. The first level corresponds to the elements close to the central region of the adhesive and the second to those

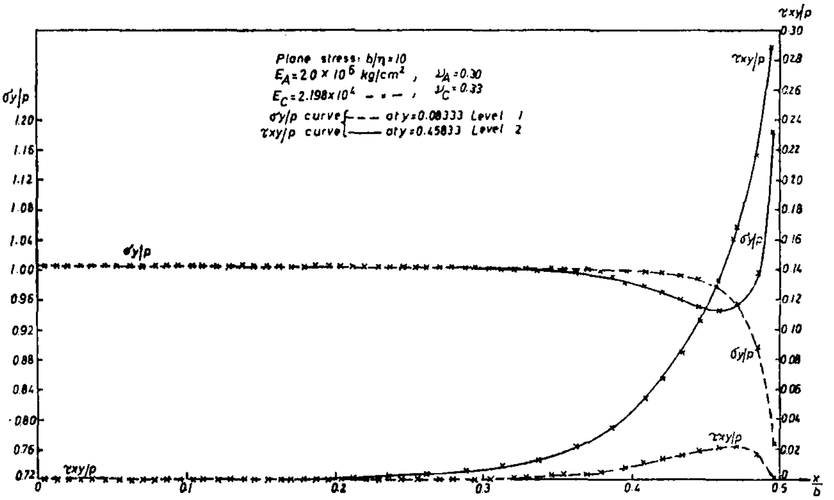

FIGURE 3 Variation of σ_y and τ_{xy} for plane stress case.

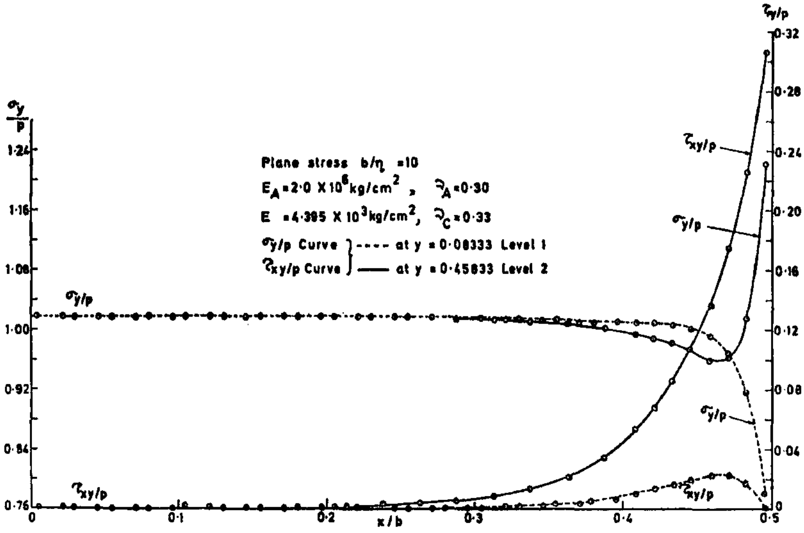

FIGURE 4 Variation of σ_y and τ_{xy} for plane stress case.

close to the interface. These two levels are referred to as level 1 and level 2 (Figure 2). Figures 3 to 6 show the distribution of σ_y and τ_{xy} for plane stress case and Figures 7 to 10 show the distribution of the corresponding adhesive stresses σ_z and τ_{RZ} for the axisymmetric case. In these figures the stresses have been non-dimensionalized by dividing them by the applied stress, p.

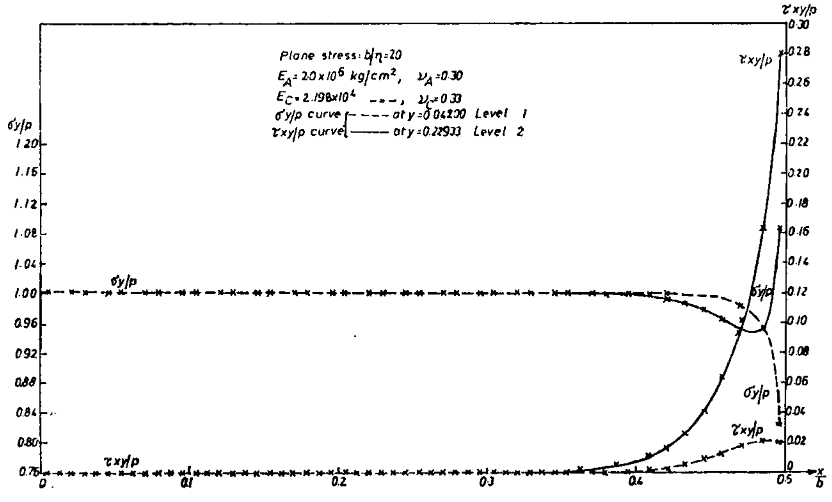

FIGURE 5 Variation of σ_y and τ_{xy} for plane stress case.

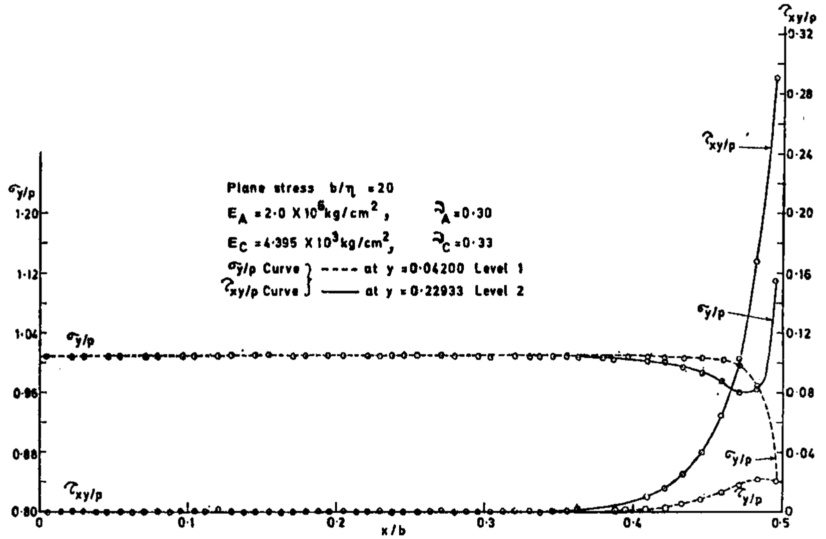

FIGURE 6 Variation of σ_y and τ_{xy} for plane stress case.

From the Figures 3 to 6 it can be seen that σ_y/p at level 1 for all the cases decreases towards the edge of the joint after being uniform for a considerable distance from the centre. For b/η equal to 10 σ_y/p is uniform up to about 80% of the joint width and for b/η equal to 20 it is uniform up to nearly 90% of the joint width. From this point it decreases rapidly towards the edge of the joint. At level 2, σ_y/p once again remains uniform but to a lesser extent and

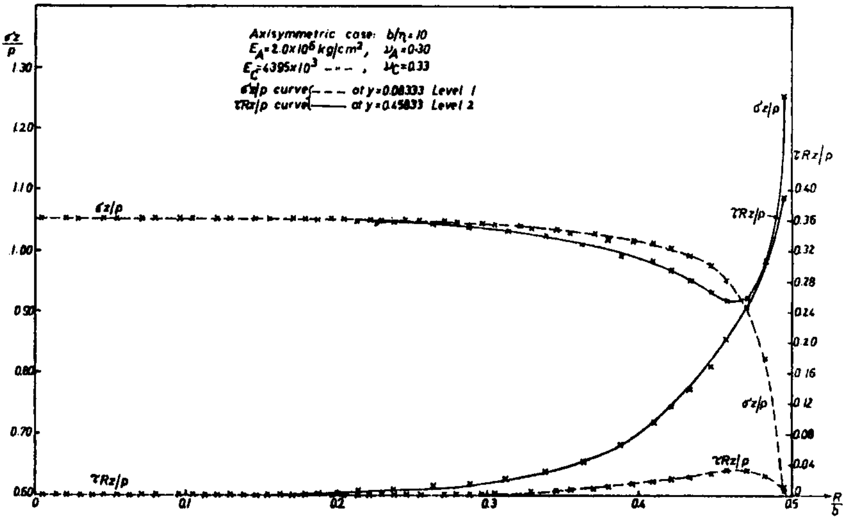

FIGURE 7 Variation of σ_z and τ_{rz} for axisymmetric case.

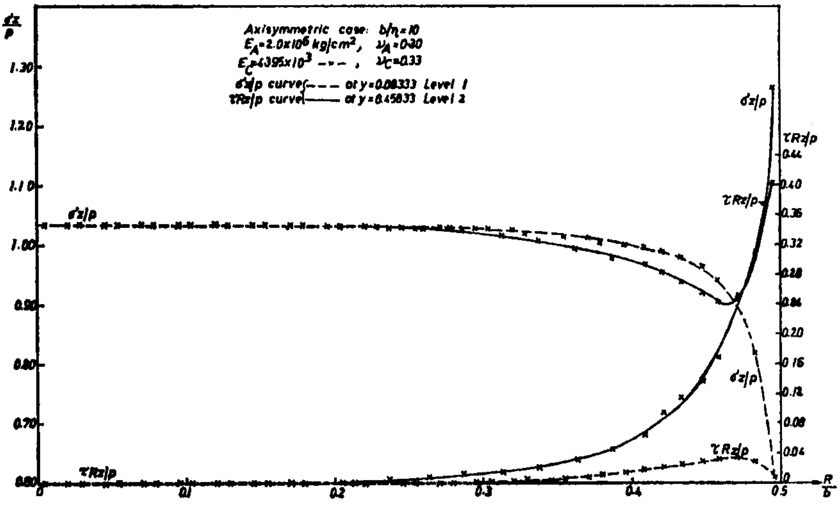

FIGURE 8 Variation of σ_z and τ_{rz} for axisymmetric case.

decreases for a short distance and then rises rapidly towards the edge of the joint indicating the stress concentration at the adhesive air interface. For b/η equal to 10, it is uniform up to about 60% of the joint width and for b/η equal to 20, it is uniform up to about 75% of the joint width, *i.e.*, to a lesser distance than at level 1.

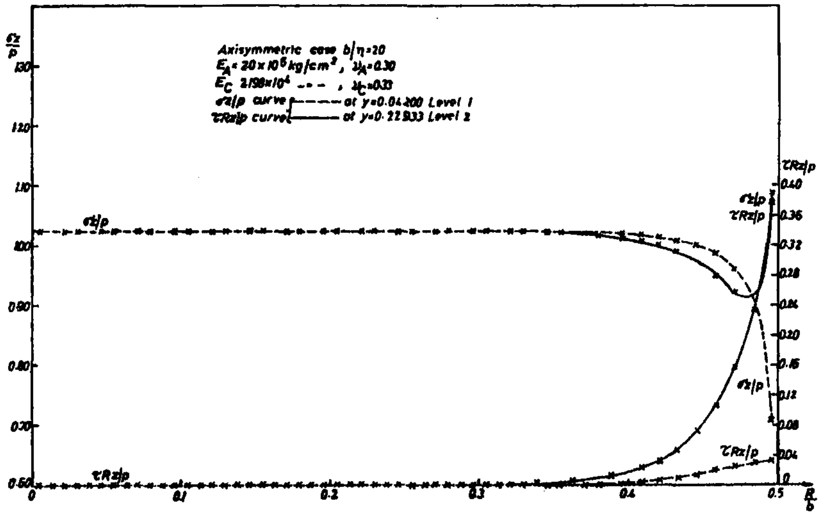

FIGURE 9 Variation of σ_z and τ_{rz} for axisymmetric case.

It is interesting to note that at level 2, σ_y/p decreases over a certain length before increasing towards the edge. A similar trend is observed in the case of squeezing of a highly viscous fluid between two rigid plates.[11] It is seen that with considerable increase in the adhesive modulus no significant change in the maximum stress concentration can be observed although there is an increase in stress concentration with decrease in E value. This may be due to the fact that the evaluation of the maximum stress is done at a point in the neighbourhood of the adhesive air interface, within the adhesive. In order to get a clear picture of the stresses exactly at the edge extrapolation of centroidal values is necessary which requires considerable judgment.

τ_{xy}/p follows the same pattern as for σ_y/p for b/η equal to 10. At level 1 the magnitude of τ_{xy}/p is quite small and after being almost zero up to 60% of the joint width it increases to a small value and decreases again to zero close to the edge. At level 2, it is close to zero up to around 40% of the joint width and subsequently rises steeply towards the edge. For b/η equal to 20 τ_{xy}/p at level 1, is zero up to about 80% of the joint width and then rises to a small value, but does not become zero. At level 2 it is zero up to about 70% of the joint width and then rises steeply towards the edge. The shear stress should reduce to zero at the edge. Once more there is no significant variation in its magnitude with change in the modulus of elasticity of the adhesive.

The analysis given in Ref. 10 is based on the assumption of constant stress through the thickness and as a result in the case of butt joints, there is zero stress concentration in σ_y which is extremely improbable taking into account the presence of stress singularity at the interface corner. In addition the shear stress was found to be zero throughout the adhesive.

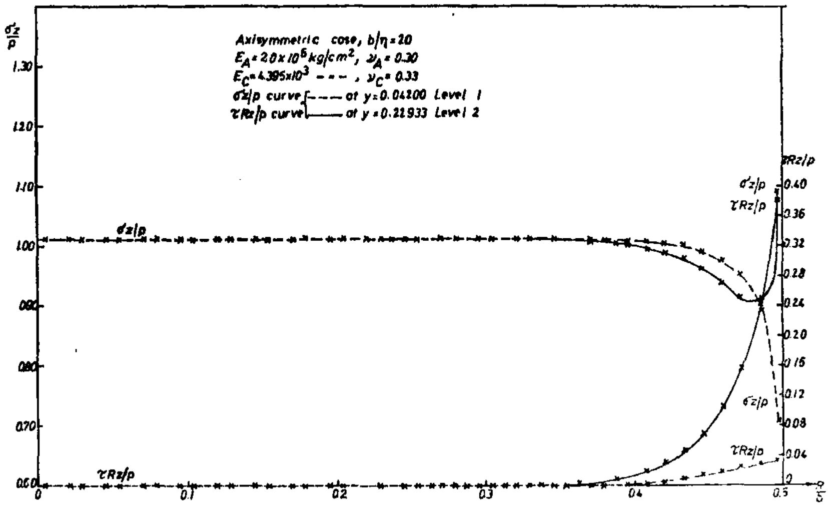

FIGURE 10 Variation of σ_z and τ_{rz} for axisymmetric case.

The behaviour under the axisymmetric case follows the same pattern as under the plane stress case. However, the stresses are uniform to a lesser distance and the stress concentrations are slightly higher.

From the above discussion it can be concluded that there is significant variation of the stress pattern across the adhesive thickness specially near the adhesive–air interface, and that the stresses are not uniform through the thickness of the adhesive.

It is a fairly well established fact that apart from the adherends and adhesive in a composite structure, there are surface layers at the interface which have characteristics different from the bulk material and these are known as boundary layers. Therefore an analysis of the mechanical behaviour of adhesive joints must give due importance to the effect of such boundary layers. The presence of boundary layers can be easily considered in the finite element analysis. Presently the effect of boundary layers on the stresses in a butt joint is under investigation.

NOMENCLATURE

b	Width of the adhesive joint
E_A	Young's modulus of the adherend
E_C	Young's modulus of the adhesive
η	Thickness of the adhesive
p	Applied stress
X, Y	Coordinate axes for plane stress case
R, Z	Coordinate axes for axisymmetric case
ν_A	Poisson's ratio for adherend
ν_C	Poisson's ratio for adhesive
σ_y, τ_{xy}	Normal and shear stresses in plane stress case
σ_z, τ_{RZ}	Normal and shear stresses in axisymmetric case

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